Toward a Profitable Grid-Connected Hybrid Electrical Energy Storage System for Residential Use

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Abstract—Hybrid electrical energy storage (HEES) systems have the potential to result in considerable cost savings by reducing the electric bills of home users. This paper first presents grid-connected dual-bank HEES system design and management to maximize the electric bill savings for residential users, and subsequently provides a comprehensive sensitivity analysis of the economic feasibility of residential HEES systems. Specifically, the paper describes a daily management policy based on energy buffering strategy with one bank as the main storage bank and the other as the energy buffering bank, and then derive the global design of HEES specifications based on the daily management results. Simulation results prove the effectiveness of energy buffering strategy and show the proposed HEES system is capable of bringing in profits under current input variables. Finally, a detailed analysis is conducted to show how each input variable affects the final design of the proposed residential HEES system and the maximum annual profits it achieves. Together with the design and control mechanism, the proposed analysis provides potential customers with the comprehensive knowledge of how HEES systems can be deployed to achieve savings in their electric bills.

Index Terms—Electric bill savings, grid-connected hybrid electrical energy storage (HEES) system, residential users, sensitivity analysis.

I. INTRODUCTION

ONE OF the greatest challenges in power grid is the mismatch between electricity generation and consumption. Electric power demand ramps up during certain hours of a day (also known as peak hours), and the load demand following from the generation side is generally expensive and/or limited [2], thus requiring electric utility over-provisioning to avoid blackouts during peak hours. To relieve requirements on the peak capacity of electricity generation, utility companies employ time-of-use pricing policy (also known as time-of-day policy) with higher electricity unit price during peak hours [3]. Besides, various real-time electricity pricing (RTP) policies [4] have been proposed. These pricing policies encourage residential users to perform spontaneous management of their electric energy usage, i.e., demand side management (DSM). DSM not only lowers the electric bills of residential users, but also reduces the demand on the peak power generation capability of utility companies.

One way to conduct DSM is electricity load scheduling, i.e., to shift some of the residential electricity load tasks from peak hours to off-peak hours [5], [6]. This method may only be partially effective: 1) a large portion of the user-specified tasks (e.g., the usage of TV and air conditioner) can hardly be shifted to other time slots and 2) some other tasks have timing limitations (e.g., washing machines may not be used during midnight due to noises). A more promising method for households is to exploit grid-connected electrical energy storage (EES) systems. They store the purchased energy from the grid when the electricity price is low and release it when the price is high, and therefore manages to transfer part of peak-hour power demand to off-peak hours.

In order to return to EES users as much profit as possible, a grid-connected EES system should achieve a reasonably low cost, high efficiency, high energy density, and a long lifetime. Conventional EES system deployments consist of a single type of energy storage element. The overall performance of a homogeneous EES system is therefore primarily determined by the underlying characteristics of the embedded energy storage element. Unfortunately, none of the existing energy storage elements can simultaneously fulfill all the desired features of a profitable EES system for residential users.

A promising method to overcome the aforesaid limitations of homogeneous EES system is to deploy hybrid EES (HEES) system [7], [8]. A HEES system is comprised of two or more heterogeneous energy storage elements, and have the potential to utilize the advantages of each energy storage type while hiding their weakness. To achieve this, a system developer must provide an optimal system design and a systematic management policy based on the requirements of the application, which may involve a combined consideration of the charge/discharge efficiency of each energy storage element,
capacity degradation, and expected service life of the HEES system, etc. The residential application of a grid-connected HEES system requires the consideration of more real-life factors besides the above-mentioned features, to name a few, the capital cost of energy storage elements and electricity pricing, making the design and management of a residential HEES system even more challenging.

In practice, some crucial input variables to the problem of residential HEES system design and management vary over time and/or space, and customers must be provided with the knowledge of maximum possible profits along with the most profitable strategies under their own conditions before purchasing a HEES system. For example, the unit price of lithium-ion (Li-ion) batteries keeps decreasing as the technology advances [9], [10], the electricity prices differ from city to city but exhibit an increasing trend over time [11], and different residential users have different amounts of available space for such a system. For a customer who lives in the area where the electricity price is $a$ per kWh in base hours and rises to $b$ times of base-hour price during peak hours, and has $c$ cubic feet available for installing the system, the first and foremost question for HEES designer is whether to choose a HEES over a homogeneous EES; and if yes, what kinds of energy storage elements and capacities. Moreover, what the optimal decision will be if the electricity price is expected to increase by $d\%$ per year going forward.

This paper demonstrates how to maximize the profits with a grid-connected dual-bank HEES system by saving electric bills for residential users. We first reason why we manage the energy storage system based on the energy buffering strategy, i.e., we adopt one type of energy storage element as the main storage bank and a second bank with another type of energy storage element to buffer the energy conversion. Then, a two-step mechanism is proposed. The first step is to calculate the maximum electric bill savings with a fixed system capacity settings, i.e., to derive the optimal daily management policy. An accurate simulator dedicated for HEES applications called SIMES [12] is employed to evaluate the actual energy saving achieved the daily optimization results. The second step is to determine the capacities of each battery bank based on the daily optimization results to maximize the annual profit, which is the total energy cost saving minus the overall system cost amortized over the system lifetime. We further conduct comprehensive sensitivity analysis and summarize how each input variable affects the economic feasibility of HEES systems. Simulation results show that the proposed HEES system achieves $118$ pure profit each year for a budget of $3000$ and system volume of $50$ L. It is also proved that the electricity rates have major effects on the profitability and design of HEES systems, e.g., if the ratio of peak-hour electricity price to base-hour price falls below a certain threshold ($3$ in our case), the HEES system cannot make profits. The capital cost of the energy buffering bank has higher influence on the HEES annual profits. The system annual profit increases by $130.19\%$ if the capital cost of the energy buffering bank is halved for a budget of $3000$ and system volume of $50$ L, compared to a $8.76\%$ increase if the capital cost of the main storage bank is halved.

The rest of this paper is organized as follows. Section II summarizes prior arts, and Section III presents the performance metrics of EES elements and electricity pricing policies. Section IV shows the HEES system setup. The two steps of HEES system design mechanism are elaborated in Sections V and VI, respectively. Section VII shows the experimental results and the sensitivity analysis in detail, and finally, Section VIII concludes this paper.

II. RELATED WORK

A. HEES System Control and Applications

The concept of hybridizing power sources is first introduced in [13]. Researchers have invested considerable efforts to explore the optimal control schemes and architectures for HEES systems. For example, Xie et al. [14] proposed a near-optimal solution to charge EES elements considering detailed EES and converter models. Yue et al. [12] designed a HEES simulator called SIMES, which has detailed models of various energy storage elements and power converters. Ge et al. [15] combined a HEES system with an energy harvesting system and maximize the delivery efficiencies of conversion circuitries during the charging process.

HEES technology has been deployed in a wide range of applications. They are deployed in electric vehicles (EVs) to replace homogeneous EES systems to improve the charge and discharge efficiencies [16]–[19]. HEES systems are also applied to microgrids [20], [21] and renewable energy systems [22]–[24] to provide energy storage as well as frequency control. Among these works, economic feasibility analysis of HEES systems applied to these applications are crucial in promoting wide adoption of HEES systems. Prodromidis and Coutelieris [23] presented an economic analysis of a battery and flywheel HEES system as stand-alone system for energy supply. Ma et al. [22] put forward an economic analysis about a HEES system with batteries and pumped hydro storage to support stand-alone renewable energy systems. The economic feasibility of HEES systems in large-scale power systems is also analyzed, in which the HEES implements frequency control to avoid breakouts [25].

B. Residential Electricity Cost Reduction

Grid-connected EES system deployment has been an emerging solution for years to encourage residential users to perform DSM as battery technologies evolve [26]–[29]. For example, Purvins et al. [27] focused on homogeneous EES system application in households and assume fixed round-trip efficiency (cycle efficiency) for their EES system. Han et al. [29] adopted a novel cooperative game model to select and size battery bank for HEES systems to implement load shifting, but without consideration of detailed battery models. The actual EES elements have cycle efficiency ranging from less than 50% to over 90% with different discharge currents and battery aging. Detailed energy storage element models are especially important for HEES system design since the actual efficiency may vary a lot with different discharge current control policies. Zhu et al. [30] presented a HEES system design and control methodology aimed for residential users to reduce electric bills, but its
management policy is unable to fully utilize the hybridized nature of HEES system.

This paper distinguishes itself from existing studies by incorporating the energy buffering strategy for HEES in a residential grid-connected HEES system to save electric bills and considering detailed EES efficiency and lifetime models. Furthermore, we provide a systematic analysis of how the bill savings from such a system are affected by electricity pricing, capital cost of EES elements, etc.

III. BACKGROUND
A. Performance Metrics of EES Elements

Popular commercial off-the-shelf EES elements include lead-acid batteries, Li-ion batteries, nickel-metal hydride (NiMH) batteries, metal-air batteries, and supercapacitors. Table I summarizes five performance metrics of these energy storage elements, all of which essential to the residential energy management system. The strengths of each type of energy storage element are highlighted in boldface and the weaknesses are in italic.

1) Cycle Efficiency: The performance of an energy storage system greatly relies on the charge/discharge efficiency of its embedded energy storage elements. The discharge efficiency of an energy storage element is defined as the ratio of its output current to the actual degradation rate of its stored charge. One of the major factors that affect the discharge efficiency of a battery is the rate capacity effect, which depicts the actual rate of internal charge loss \(I_{eq}\) is a superlinear function of its output discharge current \(I_{\text{disch}}\) [32]

\[
I_{eq} = \left( \frac{I_{\text{disch}}}{I_{\text{ref}}} \right)^k \cdot I_{\text{ref}} \tag{1}
\]

where \(I_{\text{ref}}\) is the reference discharge current. Unless explicitly provided, it is the constant current that takes 20 h to fully discharge a battery with nominal full-charge capacity (FCC) \(Q\) (in Ah), i.e., \(I_{\text{ref}} = Q/20\). The constant \(k\) reflects the actual efficiency of the discharging process. The typical \(k\) value of lead-acid batteries ranges from 1.3 to 1.4 whereas that of Li-ion batteries is less than 1.1 according to the battery characterization results in our laboratory. Supercapacitors, on the contrary, have \(k = 1\), i.e., they have no rate capacity effect.

2) Self-Discharge Effect: EES elements continuously lose stored energy even if they are disconnected from any load device. This phenomenon is known as self-discharge. Although rate capacity effect is not a concern for supercapacitors, they suffer severe self-discharge effect, whereas most batteries absorb negligible amount of self-discharge. A supercapacitor loses 20%–40% of its stored energy in one day when no load is connected [8].

3) Cycle Life and Capacity Degradation: The cycle life is defined as the number of charge/discharge cycles an energy storage element performs before its capacity drops to a specific percentage (typically 80%) of its initial FCC. In more accurate lifetime models, capacity degradation describes the fact that the effective FCC of a battery gradually drops cycle by cycle. The detailed models of the energy storage elements adopted in our method are discussed in Section VI-A.

4) Capital Cost: As a crucial part in the HEES system cost, the capital cost of energy storage elements employed in the system directly affects the system’s profitability. The unit price of an energy storage element is defined as the dollar cost per unit energy. For example, a battery’s unit price is the dollar cost of the battery divided by its nominal FCC \(Q\) (in Ah) and the terminal voltage \(V\). According to Table I, lead-acid batteries and metal-air batteries are among the cheapest types of energy storage elements, whereas Li-ion batteries and NiMH batteries have over four times the unit price. Supercapacitors have significantly higher unit prices compared to commercial batteries.

5) Energy Density: Yet another important metric of energy storage elements in a residential system is the energy density, i.e., the amount of stored energy per unit weight or volume. Li-ion batteries and metal-air batteries have higher energy density compared to lead-acid batteries, while supercapacitors have generally lower energy density than electrochemical batteries. Since residential application often has limits on the overall system size, volumetric energy density is used in this paper. We define the reciprocal of battery volumetric energy density as unit volume, calculated by the volume of a battery divided by the maximum stored energy.

### B. Electricity Pricing Policies

The deployed electricity pricing policy is a crucial input variable which not only affects the capacities of each energy storage bank but also the system management scheme, therefore having significant influence on the profitability of residential EES systems.

1) Time-of-Use Pricing Policy: Among all the prevailing time-varying pricing policies, time-of-use pricing policy is most widely adopted to incentivize users to perform DSM.\(^1\) Time-of-use pricing policies set different electricity unit price for different times of a day. For example, the pricing policy in New York City has a peak-hour period from 10 A.M. to 10 P.M., when peak electricity price is applied. In addition, utility companies apply higher peak-hour price in summer

\(^1\)In some states such as California, all business and agriculture customers are required to transition to time-of-use pricing (see http://www.pge.com/en/mybusiness/rates/tvp/toupricing.page).
(e.g., June to September in New York City) than other months. It is because during the peak hours in summer months, the power consumption from cooling (air conditioners, fans, etc.) adds up to the peak hour demand, as shown in Fig. 1. Table II lists the detailed prices adopted by Consolidated Edison, Inc. (Con Edison), in New York City.

2) Real-Time Electricity Pricing: Recent research in the Smart Grid field has proposed various RTP, in which the electricity unit price in a certain time slot is not fixed, but a function of the energy consumption in the current time slot, and this function may differ in different hours in a day [4]. More precisely, the electricity unit price increases with the increase of the energy consumption during a time slot, and/or adjusted based on current wholesale price, thereby discouraging high energy consumption. In this paper, we assume the electricity pricing functions in real-time pricing policies are known at design time. In the cases where the electricity price is not known beforehand, the proposed methodology can be combined with price prediction techniques such as [5] and [33], which predict future electricity prices to determine the daily energy management policies.

3) Pricing Policies Used in Simulation: Without loss of generality, we adopt two time-of-use pricing policies and two RTP policies in this paper, namely: 1) the Con Edison time-of-use pricing policy; 2) a finer-grained version of Con Edison pricing, as shown in Fig. 2(a); 3) a discrete two-tier price; and 4) an increasing continuous RTP function as shown in Fig. 2(b). As a form of time-of-use pricing, the second policy has linear pricing functions as 1), but it changes the electricity unit price more frequently with finer time granularity than 1), with an average peak-hour unit price equal to the peak-hour rate of Con Edison pricing. For the RTP policies, during a unit-length time slot, the discrete two-tier pricing policy 2) asks for twice the regular unit price for the part of energy that exceeds a certain threshold. The continuous pricing 4) has an increasing unit energy price: $u(E) = u_0 E^{0.6}$, where $u_0$ a constant. Note that the discrete pricing and the continuous pricing functions are set in such a way that the total cost $c(E)$ equals that of Con Edison time-of-use pricing when $E = 4 \text{ kWh}$ in an hour. The time-of-use pricing policy, except where noted, denotes the Con Edison policy hereinafter. All these pricing policies have different price settings for low season and high season.

IV. HEES System Setup

A dual-bank HEES system, i.e., an EES system equipped with two different types of EES elements, is the most popular setup for HEES system designs. Although HEES systems with three or more banks can achieve better performance than dual-bank HEES systems, the gain is much smaller than that from homogeneous EES system to dual-bank HEES system [31], especially for residential use where the demand profile contains only moderate demand spikes with the grid always connected to avoid breakouts. Moreover, a HEES system with three or more banks requires a lot more operational costs, adding to the total system cost, making it more challenging to achieve profitability [34]. Therefore, a dual-bank HEES system setup is good enough for residential uses.

An efficient management scheme for a HEES system with two heterogeneous banks is based on the energy buffering strategy as stated in detail in Section V, which includes a main energy storage bank and an energy buffer bank. Based on the performance metrics described in Section III-A, we choose lead-acid batteries as the main energy storage bank and Li-ion batteries as the energy buffer. The very first criterion in choosing the energy storage elements of the main storage bank is low capital cost, otherwise difficult for the system to make profits. Second, they should have acceptable cycle efficiency and lifetime as well. According to Table I, lead-acid batteries and metal-air batteries have very low capital cost. Although metal-air batteries are even cheaper, their cycle efficiency is significantly lower than lead-acid batteries and they also have much shorter lifetime. Therefore, we deploy lead-acid batteries as the main energy storage. On the other hand, the energy buffer bank should be able to cycle energy with as little loss as possible to achieve high performance. Supercapacitors and Li-ion batteries both have high cycle efficiency. The former, however, have extremely high capital cost under current technology. Hence, Li-ion batteries are adopted as the energy buffer.
energy cost. As a result, the maximum delivered power of lead-acid battery bank is limited and may not be sufficient for the system to perform load demand peak shaving.

Second, the system output currents must be lowered when the demand falls low. Most utility companies do not repurchase the stored electricity in EES systems. Selling energy back to the grid is hence not allowed in our problem formulation, i.e., the delivered energy of the battery banks cannot exceed load demand during a unit-length time slot. This means the lead-acid battery bank must be discharged at very low currents in low-demand time slots if its delivered energy cannot be used to charge Li-ion batteries.

We show that the load demand valleys (i.e., the time slots with low demands) may lead to efficiency degradation of the HEES system. Consider a lead-acid battery bank with 50 batteries in parallel, each with an energy capacity of 1 kWh. Fig. 4 shows a 4-h length of the lead-acid battery bank’s delivered power profile, restricted by the load profile (solid line). The ideal scheme of the lead-acid battery bank (dashed line) is to deliver constant output power, achieving highest efficiency while delivering the same amount of energy as the actual profile (dotted line). This ideal profile cannot be realized without energy buffering, because the actual lead-acid battery energy loss must be lower than the load demand if the Li-ion battery bank cannot get charged during peak hours as in [30]. The overall energy loss in the battery bank is 3.833 kWh in this 4-h duration, 8.67% more than the energy loss of the ideal profile, 3.528 kWh.

The two observations above lead to the daily energy management policy based on energy buffering. The Li-ion battery bank may get charged by the lead-acid battery bank when necessary (when the load demand is too low), and releases energy when demand peaks arrive. More precisely, during demand valleys (e.g., from 2 to 2.7 h in Fig. 4), the lead-acid battery bank provides energy to both load devices and the Li-ion battery bank. In this way, the system’s peak power generation ability is greatly improved since the Li-ion battery bank can get charged during peak hours.

**B. DCR Problem Formulation**

This section formulates the DCR problem and describes its solution. Table III lists the symbols in the rest of this paper.

1) **Objective Function:** The optimization variables are the current flows of the two battery banks during peak hours, i.e., $x_{Li}^{1}, \ldots, x_{Li}^{t2}$ and $x_{Pb}^{1}, \ldots, x_{Pb}^{t2}$, where $t1$ and $t2$ denote...
the beginning and the end decision epoch of the peak hours, respectively. These current variables are positive when the battery banks are being discharged, and negative otherwise. The objective is to maximize the daily energy cost saving, defined by the energy cost saving during peak hours subtracted by the energy cost saving during base hours. We look into the two parts separately.

1) The energy cost reduction during peak hours (P is the set of decision epochs in peak hours)

\[ \sum_{i \in P} \left( c_i^P \left( E_i^{\text{load}} \right) - c_i^P \left( E_i^{\text{load}} - \Delta E_i^{\text{load}} \right) \right) \]  

where

\[ \Delta E_i^{\text{load}} = \begin{cases} \eta_1 \cdot x_i^L \cdot V_i^{Li} + \eta_1 \cdot x_i^P \cdot V_i^{Pb} / N & \text{if } x_i^L \geq 0 \\ \eta_2 \cdot x_i^L / \eta_2 + \eta_1 \cdot x_i^P \cdot V_i^{Pb} / N & \text{if } x_i^L < 0. \end{cases} \]

\[ x_i^P \] is non-negative during peak hours as the lead-acid battery bank only discharges, whereas \[ x_i^L \] can be either negative or positive to achieve the buffering function of the Li-ion battery bank. When \[ x_i^L < 0 \], the Li-ion battery bank is charging through a ac–dc rectifier (which has an efficiency of \[ \eta_2 \] as listed in Table III), therefore the actual power the Li-ion bank requires is \[ x_i^L \cdot V_i^{Li} / \eta_2 \].

2) The additional energy cost from charging the battery banks during base hours (B is the set of decision epochs in base hours)

\[ - \sum_{i \in B} \left( c_i^B \cdot x_{i0}^{Li} \cdot V_i^{Li} + x_i^{Pb} \cdot V_i^{Pb} / \eta_2 \right) \]

where the constant charging currents \[ x_{i0}^{Li} \] and \[ x_i^{Pb} \] are calculated by

\[ x_{i0}^{Li} = \frac{1}{24-t_2/N + (t_1-1)/N} \sum_{i \in B} \Delta Q_i^{Li} \]

\[ x_i^{Pb} = \frac{1}{24-t_2/N + (t_1-1)/N} \sum_{i \in B} \Delta Q_i^{Pb}. \]

\[ (24-t_2/N + (t_1-1)/N) \] is the length of base hours. Since the lead-acid battery bank is always discharging during peak hours, \[ \Delta Q_i^{Pb} \] is positive for any \( i \), calculated by

\[ \Delta Q_i^{Pb} = \left( \frac{x_i^{Pb}}{\bar{I}_i^{Pb}} \right) \cdot \frac{I_{i0}^{Pb}}{N} = \left( \frac{20x_i^{Pb}}{Q_i^{Pb}} \right) \cdot \frac{Q_i^{Pb}}{20} / N \]

where \[ \bar{I}_i^{Pb} = (Q_i^{Pb}/20) \] denotes the reference current for lead-acid batteries. \[ \Delta Q_i^{Li} \] for Li-ion batteries can be either positive or negative. It is calculated by

\[ \Delta Q_i^{Li} = \begin{cases} \frac{x_i^{Li}}{\bar{I}_i^{Li}} \cdot I_{i0}^{Li} / N & \text{if } x_i^{Li} \geq 0 \\ \frac{x_i^{Li}}{I_{i0}^{Li}} / N & \text{if } x_i^{Li} < 0. \end{cases} \]

2) System Constraints: The HEES system control scheme must satisfy three constraints, namely the load demand constraint, the Li-ion battery capacity constraint, and the lead-acid battery capacity constraint. The load demand constraint implies that the energy provided by the HEES system at any time slot cannot exceed the load demand, i.e., \[ E_i^{\text{load}} \geq \Delta E_i^{\text{load}}, \forall i \in P \].

The Li-ion battery capacity constraint says that the amount of charge stored in the Li-ion battery bank should be, at any time, within the range from 0 to \[ Q_i^{Li} \]. We first calculate the remaining charge at the end of \( i \)th time slot by \[ \sum_{i \in P} \Delta Q_i^{Li} - \sum_{l=1}^i \Delta Q_l^{Li} \], where \[ \sum_{i \in P} \Delta Q_i^{Li} \] is the initial charge stored in the Li-ion battery bank at the beginning of peak hours. Therefore

\[ 0 \leq \sum_{i \in P} \Delta Q_i^{Li} - \sum_{i=1}^l \Delta Q_i^{Li} \leq Q_i^{Li}, \forall l \in P. \]

The lead-acid battery bank’s remaining charge should also satisfy the capacity constraint, but not as complex as the Li-ion battery constraint. Since the lead-acid battery bank can only be discharged \( (x_i^{Pb} \geq 0) \) during peak hours, we only need the
total charge loss over the peak hours to be no more than its capacity $Q_{\text{Pb}}$.

3) **Problem Formulation and Solution:** The DCR problem is therefore formulated as follows.

**Given:**
1) Battery bank capacities (in Ah) $Q_{\text{Li}}, Q_{\text{Pb}}$.
2) Battery terminal voltage $V_{\text{Li}}, V_{\text{Pb}}$.
3) The peak-hour energy cost function $c_i^p(E)$.
4) The base-hour unit energy price $u_{\text{B}}$.
5) Residential load energy profile $E_{i,\text{load}}$, $i = 1, \ldots, 24N$.
6) Batteries’ rate capacity effect coefficients $k_{\text{Li}}, k_{\text{Pb}}$.
7) DC–AC inverters’ power conversion efficiency $\eta_1$ and ac–dc rectifier’s efficiency $\eta_2$.

**Find:** Discharge current profiles $x_{i,\text{dis}}^{\text{Li}}$, $x_{i,\text{dis}}^{\text{Pb}}$ and $x_{i,\text{use}}^{\text{Li}}$, $x_{i,\text{use}}^{\text{Pb}}$.

**Maximize:** The daily energy cost reduction

$$
\text{DCR}(x_{i,\text{dis}}^{\text{Li}}, \ldots, x_{i,\text{use}}^{\text{Li}}, x_{i,\text{dis}}^{\text{Pb}}, \ldots, x_{i,\text{use}}^{\text{Pb}}) = \sum_{i \in \mathbf{P}} (c_i^p(\Delta E_{i,\text{load}}) - \sum_{i \in \mathbf{B}} (u_{\text{B}} \cdot \frac{x_{i,\text{use}}^{\text{Li}}, V_{\text{Li}} + x_{i,\text{dis}}^{\text{Pb}}, V_{\text{Pb}}}{\eta_2}))
$$

(10)

where $\Delta E_{i,\text{load}}$, $x_{i,\text{dis}}^{\text{Li}}$, and $x_{i,\text{dis}}^{\text{Pb}}$ are given by (3), (5), and (6).

**Subject to:**
1) The load energy constraints

$$E_{i,\text{load}} - \Delta E_{i,\text{load}} \geq 0, \forall i \in \mathbf{P}. \quad (11)$$

2) The Li-ion battery capacity constraints

$$0 \leq \sum_{i \in \mathbf{P}} \Delta Q_{i,\text{Li}} \leq Q_{\text{Li}}, \forall i \in \mathbf{P} \quad (12a)$$

$$0 \leq \sum_{i \in \mathbf{P}} \Delta Q_{i,\text{Li}} - \sum_{i = i_1}^l \Delta Q_{i,\text{Li}} \leq Q_{\text{Li}}, \forall l \in \mathbf{P}. \quad (12b)$$

3) The lead-acid battery capacity constraints

$$x_{i,\text{dis}}^{\text{Pb}} \geq 0, \forall i \in \mathbf{P} \quad (13a)$$

$$\sum_{i \in \mathbf{P}} \Delta Q_{i,\text{Pb}} \leq Q_{\text{Pb}}, \forall i \in \mathbf{P}. \quad (13b)$$

The DCR problem is a nonconvex optimization problem. We adopt a heuristic to solve this problem efficiently. Because energy reduction $\Delta E_{i,\text{load}}$ and Li-ion battery charge loss $\Delta Q_{i,\text{Li}}$ are discontinuous at $x_{i,\text{dis}}^{\text{Li}} = 0$ as shown in (3) and (8), we first determine the time slots when the Li-ion battery bank is charged to remove the discontinuity, i.e., determine the indices of negative $x_{i,\text{dis}}^{\text{Li}}$ variables. Specifically, we assume the lead-acid battery bank is discharged using a constant current $x_{i,\text{dis}}^{\text{Pb}}$ (this value is introduced merely to determine the signs of $x_{i,\text{dis}}^{\text{Li}}$). At the $i$th time slot, if the energy provided by the lead-acid battery bank $\eta_1 \cdot x_{i,\text{dis}}^{\text{Pb}}, V_{\text{Pb}}/N$ is smaller than the energy required by the load $E_{i,\text{load}}$, the Li-ion battery bank must be discharged, which means $x_{i,\text{dis}}^{\text{Li}} \geq 0$, otherwise (i.e., if the energy provided by the lead-acid battery bank is larger than the energy required by the load) $x_{i,\text{dis}}^{\text{Li}} < 0$.

The initial value of $x_{i,\text{dis}}^{\text{Pb}}$ is obtained by solving the equation: $Q_{\text{Pb}} = (((t_2 - t_1 + 1)/N) - ((20x_{\text{dis}}^{\text{Pb}}, V_{\text{Pb}}^2)/(Q_{\text{Pb}})/(Q_{\text{Pb}})/20))$, the constant current that makes the lead-acid battery bank completely discharged at the end of peak hours. Therefore, the sign of $x_{i,\text{dis}}^{\text{Li}}$, i.e., whether the Li-ion battery bank is charging or discharging during the $i$th time slot, is determined by

$$
\begin{cases}
    x_{i,\text{dis}}^{\text{Li}} \geq 0 & \text{if } \eta_1 \cdot x_{i,\text{dis}}^{\text{Pb}}, V_{\text{Pb}}/N \leq E_{i,\text{load}} \\
    x_{i,\text{dis}}^{\text{Li}} < 0 & \text{otherwise.}
\end{cases}
$$

(14)

Once the signs of $x_{i,\text{dis}}^{\text{Li}}$ values are determined, the solution is derived by the MATLAB optimization fmincon. We then increase $x_{i,\text{dis}}^{\text{Pb}}$ by a small amount several times. In each iteration, signs of $x_{i,\text{dis}}^{\text{Li}}$ values are determined by the current $x_{i,\text{dis}}^{\text{Pb}}$ value and the problem is then solved by fmincon. We record the best solution with the maximum energy cost saving during the procedure.

The daily energy cost saving achieved by the optimal current control policy on the $j$th day is a function of the Li-ion battery bank capacity $Q_{\text{Li}}$ and the lead-acid battery bank $Q_{\text{Pb}}$, denoted by $f_j(Q_{\text{Li}}, Q_{\text{Pb}})$ hereinafter. The daily energy saving results under different battery capacities are stored in a 2-D look-up table (LUT) with $Q_{\text{Li}}$ and $Q_{\text{Pb}}$ as indices. The global design step uses this LUT to further solve the optimal system design, as discussed in detail in the following sections.

C. **Lifetime-Aware DCR Problem**

The optimal solution of the above DCR problem assumes that the HEES system is allowed to use the full capacity range of both battery banks during base and peak hours, respectively. Nevertheless, fully charging and then discharging of batteries result in a fast capacity degradation rate, and thereby, significantly shorten the battery lifetime [35]–[37]. On the other hand, allowing only part of the total capacities to be used for storage seems to “waste” the unused part of bank capacity, but may gain more benefits from the extended battery lifetime. Therefore, we reconsider the DCR problem based on the lifetime characterization of Li-ion batteries and lead-acid batteries. Instead of allowing full-capacity-range charging/discharging, we set the usable capacity upper limit to $s_{\text{Li}}$ of the full Li-ion battery capacity and $s_{\text{Pb}}$ of the full lead-acid battery capacity ($s_{\text{Li}}, s_{\text{Pb}} \in (0, 1)$).

We adopt the lifetime model proposed in [35] for Li-ion batteries, which proves that the lifetime of a Li-ion battery gets extended superlinearly by lowering the average state-of-charge (SoC) and/or minimizing the SoC swing in each cycle. The management policy should hence limit the Li-ion battery bank to operate within the SoC range from 0 to $s_{\text{Li}}$. We adopt the Ah-Throughput model described in [38] for lead-acid batteries. It assumes that a fixed amount of energy is cycled by a lead-acid battery throughout its lifetime, which is coherent with the inversely proportional relationship between the total cycle numbers and the average depth-of-discharge (DoD). If the maximum DoD of a battery cycle is set to be 0.7, the battery is discharged from 100% to no less than 30% SoC. Fig. 5 shows Li-ion battery lifetime as a function of maximum SoC swing $s_{\text{Li}}$ and lead-acid battery lifetime as a function of maximum DoD $s_{\text{Pb}}$.

The proposed HEES system adopts lifetime-aware daily management scheme, i.e., daily usage limited to $s_{\text{Li}}$ and $s_{\text{Pb}}$.
of overall battery capacity. Let \( \hat{f}_j(Q^{L_i}, Q^{P_b}, s^{L_i}, s^{P_b}) \) denote the maximum daily energy cost reduction of the \( j \)th day achieved by solving the lifetime-aware DCR problem. The lifetime-aware DCR problem can be solved by the same heuristic as solving the lifetime-aware DCR problem. The lifetime-aware scheme adds two more indices \( s^{L_i} \) and \( s^{P_b} \) to the results of DCR problem, requiring a four-dimensional LUT which is too costly in practice. Nevertheless, if the maximum daily cost saving \( \hat{f}_j \) can be replaced by the original DCR result \( \hat{f}_j \), which is a function of two variables only, we can reduce the storage space to a 2-D LUT and solve the above issue. We must find a \( \hat{f}_j \) which does not exceed the actual value of \( \hat{f}_j \) to avoid overestimation of the actual daily cost savings of the lifetime-aware scheme.

We show that \( \hat{f}_j(s^{L_i}, Q^{L_i}, s^{P_b}) \) is a proper approximation of \( \hat{f}_j(Q^{L_i}, Q^{P_b}, s^{L_i}, s^{P_b}) \) with no overestimation, i.e.,

\[
\hat{f}_j(Q^{L_i}, Q^{P_b}, s^{L_i}, s^{P_b}) \geq \hat{f}_j(s^{L_i}, Q^{L_i}, s^{P_b}) \cdot (s^{P_b})^{1/2} \cdot Q^{P_b}.
\]

Proof: Let \( x^{L_i}_1, \ldots, x^{L_i}_t, x^{P_b}_1, \ldots, x^{P_b}_t \) be the optimal solution to the original DCR problem given Li-ion and lead-acid capacity equal to \( s^{L_i} \cdot Q^{L_i} \) and \( s^{P_b} \cdot (s^{P_b})^{1/2} \cdot Q^{P_b} \), respectively. Let

\[
g(x^{L_i}_j) = \begin{cases} 
  x^{L_i}_j & \text{if } x^{L_i}_j \geq 0 \\
  (s^{L_i})^{1/2} - x^{L_i}_j & \text{if } x^{L_i}_j < 0.
\end{cases}
\]

If we replace \( x^{L_i}_j \) with \( g(x^{L_i}_j) \) in the objective function DCR in (10), the same amount of energy is provided to the load while less charge is taken from the grid [because \( (s^{L_i})^{1/2} - x^{L_i}_j < 1 \)], meaning the daily saving is increased by replacing \( x^{L_i}_j \) with \( g(x^{L_i}_j) \), that is

\[
DCR\left(g(x^{L_i}_1), \ldots, g(x^{L_i}_t), x^{P_b}_1, \ldots, x^{P_b}_t\right) \geq DCR\left(x^{L_i}_1, \ldots, x^{L_i}_t, x^{P_b}_1, \ldots, x^{P_b}_t\right).
\]

Furthermore, \( g(x^{L_i}_1), \ldots, g(x^{L_i}_t), x^{P_b}_1, \ldots, x^{P_b}_t \) satisfy the constraints of the lifetime-aware DCR problem with inputs \( (Q^{L_i}, Q^{P_b}, s^{L_i}, s^{P_b}) \). This means that they are a set of feasible solutions of the lifetime-aware DCR problem, and thus their DCR result must be no more than the result of optimal solution \( \hat{f}_j(Q^{L_i}, Q^{P_b}, s^{L_i}, s^{P_b}) \), that is

\[
\hat{f}_j(Q^{L_i}, Q^{P_b}, s^{L_i}, s^{P_b}) \geq DCR\left(g(x^{L_i}_1), \ldots, g(x^{L_i}_t), x^{P_b}_1, \ldots, x^{P_b}_t\right).
\]

Combining (16) and (17), we have

\[
\hat{f}_j(Q^{L_i}, Q^{P_b}, s^{L_i}, s^{P_b}) \geq \hat{f}_j(s^{L_i}, Q^{L_i}, s^{P_b}) \cdot (s^{P_b})^{1/2} \cdot Q^{P_b}.
\]

This proof indicates that the actual energy reduction is underestimated by the original 2-D solution. We randomly select several days and solve the complete version of lifetime-aware DCR problem, and find that the above estimation has less than 3.4% error.

D. Evaluation of the DCR Results

In exploring an optimal current control policy, it is not practical for system designers to consider all the details of the battery model or conversion circuitry model in the numerical problem formulation, and (10) serves as a good estimation of the daily energy saving just for deriving the daily management scheme. However, once the optimal currents \( x^{L_i}_1, \ldots, x^{L_i}_t, x^{P_b}_1, \ldots, x^{P_b}_t \) are derived, it is feasible for the system to adopt a more detailed model to evaluate the energy cost savings achieved by the optimization results.

We use SIMES, the first simulation platform for HEES systems [12], to conduct the evaluation. SIMES implements a set of models for common components such as energy storage elements and conversion circuits, and is extensible for user-defined power source, load and storage elements. Compared to the objective function in (10) which does not consider battery bank’s internal resistance and assumes fixed terminal voltage, SIMES simulation regards them all as functions of the battery’s SoC, which is the case in practice. The accuracy of SIMES has been proved by actual measurement results on a HEES system prototype. Therefore in practice, we use the optimization results \( x^{L_i}_1, \ldots, x^{L_i}_t, x^{P_b}_1, \ldots, x^{P_b}_t \) of the DCR problem (the charge/discharge currents of battery banks) along with \( Q^{L_i}, Q^{P_b}, s^{L_i}, s^{P_b} \) as inputs to SIMES and obtain the energy savings based on the load power profile calculated by SIMES, instead of using (10) in the DCR formulation to calculate the daily energy cost reductions \( f_j(Q^{L_i}, Q^{P_b}, s^{L_i}, s^{P_b}) \).

By summing up the daily results, we get the maximum seasonal cost reduction function, \( \hat{f}_L(Q^{L_i}, Q^{P_b}, s^{L_i}, s^{P_b}) \) for low season and \( \hat{f}_H(Q^{L_i}, Q^{P_b}, s^{L_i}, s^{P_b}) \) for high season, by summing up the corresponding days, where \( s^{L_i} \) and \( s^{P_b} \) are the capacity limits in low season, and \( s^{L_i}_H \) and \( s^{P_b}_H \) in high season. The results are stored in two 2-D LUTs with battery capacities and capacity limits as indices. These LUTs are used to determine the specifications of a residential HEES system, i.e., before actually acquiring the system. In other words, system manufactures provide the LUTs and residential users only need to use the LUTs once (at the time of purchasing the system). After installation, the residential system only needs to solve one DCR problem once a day with present capacities \( Q^{L_i} \) and \( Q^{P_b} \) and capacity limits \( s^{L_i}_H, s^{L_i}_L \) or \( s^{P_b}_H, s^{P_b}_L \) as inputs, of which the runtime is in the scale of several seconds.

VI. GLOBAL DESIGN

This section discusses the second step of the proposed design and management method. It further determines the
optimal design specification of the HEES system, i.e., the capacities \( Q^\text{Li} \) and \( Q^\text{Pb} \) and capacity limits \( s^\text{Li} \), \( s^\text{Pb} \), \( s^\text{H} \), and \( s^\text{H}^\text{Li} \) of both battery banks, in order to maximize the amortized annual profit, referred to as the annual-profit-maximization (APM) problem.

A. Battery Capacity Degradation Model

To take into consideration the capacity degradation to improve the accuracy of our methods, we define \( \Delta L \) to be the FCC loss percentage for a cycle-aged battery, i.e., after some number of cycles, the current effective FCC of the battery equals to \( (1 - \Delta L) \) of its nominal FCC. We assume constant degradation rate based on the Ah-throughput model of lead-acid batteries, i.e., \( \Delta L^\text{Pb} \) is a linear function of the energy that the battery has cycled \( E_{\text{cycled}} \) and the total energy it can cycle throughout its lifetime \( E_{\text{total}} \). When the battery has lost 20% of its total capacity (when \( \Delta L^\text{Pb} = 0.2 \)), we consider that this battery has reached the end of its lifetime and the total energy it has cycled reaches \( E_{\text{total}} \). Therefore, at any time during its lifetime when a battery has cycled \( E_{\text{cycled}} \), the percentage of its lost capacity \( \Delta L^\text{Pb} \) is calculated by

\[
\Delta L^\text{Pb} = 0.2 \cdot E_{\text{cycled}} / E_{\text{total}}.
\]

(19)

As for Li-ion batteries, the FCC degradation percentage \( \Delta L^\text{Li} \) is a function of the number of finished charge/discharge cycles and the capacity limit \( s^\text{Li} \) as described in Section V-C. According to the energy buffering strategy, the lead-acid battery bank experiences one charge/discharge cycle per day, whereas the number of cycles of the Li-ion battery bank depends on the optimal control results of the DCR problem. We estimate the equivalent number of cycles in Li-ion batteries per day \( w \) by adding up the absolute values of Li-ion battery bank charge/discharge current flow, assuming that Li-ion battery bank is fully charged at the beginning of peak hours, that is

\[
w = \frac{1}{s^\text{Li}} \cdot \sum_{i \in \text{Pb, } d^\text{Li}_i \geq 0} \Delta Q^\text{Li}_i.
\]

(20)
The Li-ion battery capacity degradation \( \Delta L^\text{Li} \) is therefore calculated cycle by cycle based on the model in [35].

The calendar life of Li-ion battery bank must also be considered since our optimization framework allows very low usage of batteries. The calendar life of a Li-ion battery describes the capacity degradation as a result of the passage of time, modeled by [39]

\[
d = e^{6661/T - 14} (\Delta L^\text{Li}_c)^2 + e^{4437/T - 11.6} \Delta L^\text{Li}_c
\]

(21)

where \( d \) is time in days and \( T \) is temperature in Kelvin. This equation demonstrates that a Li-ion battery experiences a minimum capacity loss percentage of \( \Delta L^\text{Li} \) after \( d \) days regardless of its usage. Combining with \( \Delta L^\text{Li} \) predicted by Broussely et al. [39], we have

\[
\Delta L^\text{Li} = \max \left\{ \Delta L^\text{Li}_s, \Delta L^\text{Li}_c \right\}.
\]

(22)

B. Problem Formulation

We introduce several crucial real-life factors in the APM problem to provide a practical and accurate system design.

1) Maintenance fee occurs at the initial installation of the HEES system and at the times when one battery bank reaches its end-of-life and needs to be replaced in order to keep the HEES system operational.

2) Discount factor \( \gamma \) reflects the time value of money. In this paper, we use the annual percentage yield of a five-year certificate of deposit (2%) as reference, i.e., \( \gamma = 1 / (1 + 2\%) = 0.9804 \).

3) System form factor must be considered when the HEES system is applied to residential usage. To be more specific, the overall system should not take too much volume to fit into dwelling units.

Taking all the aforesaid factors into consideration, the APM problem is formulated as follows.

Given:

1) LUTs of the DCR results for high season and low season.

2) Unit prices of Li-ion/lead-acid batteries \( p^\text{Li}, p^\text{Pb} \).

3) Unit volumes of Li-ion/lead-acid batteries \( v^\text{Li}, v^\text{Pb} \).

4) One-time maintenance fee \( M \) and discount factor \( \gamma \).

5) Initial investment \( I \) and system’s total volume limit \( V \).

Find:

- Li-ion battery capacities and capacity limits \( Q^\text{Li}, Q^\text{Pb}, s^\text{Li}, s^\text{Pb}, s^\text{H}, s^\text{H}^\text{Li} \).
- Pb.

Maximize: Amortized annual profit \( A \).

Subject to: Budget constraint \( Q^\text{Li} \cdot p^\text{Li} + Q^\text{Pb} \cdot p^\text{Pb} + M \leq I \) and system volume constraint \( Q^\text{Li} \cdot v^\text{Li} + Q^\text{Pb} \cdot v^\text{Pb} \leq V \).

The annual profit \( A \) is the net profit that the HEES system makes. More precisely, we consider the residential HEES system as a long-term investment, of which the profit is calculated by: 1) the annual cost reduction of electric bills subtracted by 2) the initial purchase cost of the HEES system and 3) the replacement cost (including the maintenance fee and the new bank’s cost).

To derive the maximum return on investment, we assume that the HEES system operates for \( L_{\text{sys}} \) years. Let \( P_i \) denote the accumulative profit from the beginning to the \( i \)th year. Whenever a battery bank reaches its end-of-life, it is replaced with a new one and the corresponding expenditure is subtracted from \( P_i \). Therefore

\[
P_i = \frac{P_{i-1}}{\gamma} - R^\text{Li}_i \cdot (Q^\text{Li} \cdot p^\text{Li} + M) - R^\text{Pb}_i \cdot (Q^\text{Pb} \cdot p^\text{Pb} + M)
\]

\[
+ \hat{F}_H (Q^\text{Li}_i, Q^\text{Pb}_i, s^\text{Li}_i, s^\text{Pb}_i) + \hat{F}_L (Q^\text{Li}_i, Q^\text{Pb}_i, s^\text{Li}_i, s^\text{Pb}_i)
\]

(23)

where \( R^\text{Li}_i = 1 \) (or \( R^\text{Pb}_i = 1 \)) if the Li-ion battery bank (or lead-acid battery bank) is replaced in the \( i \)th year and \( R^\text{Li}_i = 0 \) (or \( R^\text{Pb}_i = 0 \)) otherwise. \( Q^\text{Li}_i \) and \( Q^\text{Pb}_i \) are degradation-aware capacities, i.e., the actual capacities of cycle-aged batteries in the \( i \)th year of the system. \( Q^\text{Li}_i \) and \( Q^\text{Pb}_i \) are updated each year according to the capacity degradation model described in Section VI-A. Therefore, the annual profit \( A \) is
calculated by
\[ A + A \cdot \gamma^{-1} + \ldots + A \cdot \gamma^{-L_{\text{sys}} + 1} = P_{\text{sys}} \]  
\[ A = P_{\text{sys}} \cdot \frac{1 - \gamma^{-1}}{1 - \gamma^{-L_{\text{sys}}}}. \]  

The value of the amortized annual profit \( A \) converges as \( L_{\text{sys}} \) approaches infinity.

In order to calculate \( A \), we need to determine when each battery bank should be replaced, i.e., the values \( R_{\text{Li}} \) and \( R_{\text{Pb}} \). Unfortunately, the Li-ion capacity degradation model requires day-by-day recursive calculation to determine \( R_{\text{Li}} \), \( R_{\text{Pb}} \), and the effective FCC in each day, making it infeasible to derive an analytical expression of its lifetime. In addition, the results of the DCR problems are stored in LUTs. Therefore, it is impractical to derive an analytical solution expressed in closed-form equations to the global optimization problems, necessitating an exhaustive search-based algorithm to find the optimal solution. As this calculation occurs only once for one HEES design, the long runtime is acceptable and an optimal solution is desirable to achieve the highest profits. Hence, we use a search-based algorithm to solve this APM problem to ensure the final design is optimal.

VII. EXPERIMENTAL RESULTS
A. DCR Problem Evaluation Results

As mentioned in Section V-D, we deploy SIMES, the accurate and efficient HEES system simulator, to evaluate the daily energy cost savings. Table IV lists primary battery parameters deployed in the SIMES simulation.

The effectiveness of the energy buffering scheme is proved in Fig. 6, which shows the output power curves of no-buff HEES system and the buffered HEES system during the peak hours in one day. Both HEES systems are comprised of a lead-acid battery bank with 4 kAh capacity and a Li-ion battery bank with 200 Ah capacity. Without energy buffering scheme, the discharge currents of the lead-acid battery bank (Pb_NoBuff) are greatly limited by the load profile, and the Li-ion battery bank gets discharged at very small rates (Li_NoBuff), its ability to cut high load peaks significantly restrained. On the contrary, the HEES system with energy buffering allows the Li-ion battery bank to recycle the energy to alleviate load peaks (Li_Buff), and as a result, the output currents of the lead-acid battery bank fluctuate at a much smaller range (Pb_Buff). The cost saving is improved by 6.10% after the introduction of the energy buffering strategy for the peak-hour load profile shown in Fig. 6.

Fig. 7 shows the seasonal cost reduction results under time-of-use pricing policy and a multifamily unit electricity usage profile in a year, based on the time-of-use pricing listed in Table II. The maximum seasonal energy cost reduction has diminishing marginal gain as the battery capacities increase.

B. APM Problem Results

This section shows the results of the APM problem of four different systems: 1) a Li-ion battery-only EES system; 2) a lead-acid battery-only EES system; 3) a HEES system with no energy buffering scheme (NoBuff); and 4) a HEES system with buffering deployed. We test two Li-ion battery models, namely the 18650 model (with nickel-cobalt-aluminum cathodes) and the Li-ion phosphate (LiFePO4 or LFP) battery. The 18650 format is among the most popular cylindrical Li-ion cells, and LFP batteries are a newly-developed Li-ion battery model with LiFePO4 as cathode material with smaller energy density but lower capital cost compared to more popular LiCoO2-based cells such as 18650, as shown in the battery specifications listed in Table V. The annual profits with different budget and volume constraints are shown in Fig. 8. Comparing the HEES system with 18650 in Fig. 8(a) and the HEES system with LFP in Fig. 8(b), we notice that the latter achieves higher annual profits mainly because of the lower capital cost of LFP models. But the HEES system with LFP model does not necessarily have significant increase in annual profits over the HEES system with 18650 model. This is due to the system volume restriction: The energy density of the LFP model is much lower than that of 18650 model. As shown in the (3000, 1000, 500)
50) result in Fig. 8(b), the maximum annual profit is achieved with only Li-ion batteries due to the volume constraint.

Table VI shows the annual profits achieved by the proposed HEES system design under the aforementioned four pricing policies, namely Con Edison time-of-use policy ("Con Edison"), the finer-grained time-of-use policy ("finer-grained"), the discrete two-tier pricing ("two-tier"), and the continuous increasing pricing function ("superlinear") as shown in Fig. 2. The budget and volume limits are $3000 and 50 L, respectively. The results show that the proposed HEES system is able to achieve higher annual profits under real-time pricing policies. This is because the real-time pricing policies have higher penalties for peak power consumptions than the Con Edison time-of-use policy with flat rates in peak hours. More precisely, in real-time pricing policies, the unit energy cost in a certain time slot increases if the energy consumption increases, whereas in time-of-use policies the unit energy cost is constant regardless of energy consumption. The changing unit energy cost in real-time policies prompts the proposed HEES system to fully utilize the energy buffering scheme to adjust the power extracted from the grid, therefore leading to more energy cost savings than the system under time-of-use policies.

### C. Sensitivity Analysis

We present the detailed sensitivity analysis in this section from four aspects, namely: 1) the electricity pricing policy; 2) the capital cost of energy storage elements; 3) the discretization in bank sizing; and 4) the accuracy of battery models. We study how these aspects affect the battery capacities and the profitability of the optimal system design.

1) Impact of Electricity Pricing Policies: Since the actual profit of the HEES system comes from the electricity price difference of the peak hours and base hours, electricity pricing policy is the most important factor in evaluating the HEES system design. In this paper, we conduct the sensitivity analysis with regard to changing pricing policies on the Con Edison time-of-use pricing since: 1) the Con Edison pricing is representative of all the policies aiming at peak-load reduction and 2) the other pricing policies have so many variables that a sensitivity analysis on all of them is both impractical and meaningless.

We analyze how the Con Edison pricing policy affect the daily management and global design as follows. As mentioned above, the Con Edison pricing deploys flat rates in both peak hours and base hours, i.e., $c^P(E) = u^P \cdot E$ and $c^B(E) = u^B \cdot E$, where $u^P$ and $u^B$ is constant. Let $\alpha$ denote the ratio of peak-hour rate to base-hour rate ($\alpha = u^P / u^B$). According to (10), the overall saving of one day is calculated by

$$DCR = \sum_{i \in P} u^P \cdot \Delta E^i_{load} - \sum_{i \in B} \left( u^B \cdot \frac{x^{Li} \cdot Li^{\eta} + x^{Pb} \cdot Pb^{\eta}}{\eta_2} \right) \tag{26}$$

Note that $\Delta E^i_{load}, x^{Li}_{0}, x^{Pb}_{0}$ are functions of daily optimization variables $x^{Li}_{1}, \ldots, x^{Li}_{2}$ and $x^{Pb}_{1}, \ldots, x^{Pb}_{2}$. Therefore, the daily management decisions are not affected by the variation of $u^B$, but dependent upon $\alpha$, the ratio of peak-hour rate to base-hour rate. As seen in (3), delivered energy $\Delta E^i_{load}$ increases as the discharge currents increase, but also raising the charging cost according to (5) and (6). This equation shows that with higher peak-to-base price ratio $\alpha$, the system can store more energy during base hours with less charging cost and deliver the energy during peak hours to have a higher energy saving, which is consistent with intuitive thoughts.

Fig. 9 shows the maximum annual profit as a function of base-hour unit price $u^B$ [Fig. 9(a)] and normalized peak and off-peak price ratio $\hat{\alpha}$ [Fig. 9(b)], under different budget and volumetric constraints. Note that in Fig. 9(b) we introduce normalized peak-to-base ratio $\hat{\alpha}$ to the original ratio in Con Edison time-of-use policy $\alpha_0$. We scale $\alpha$ in both seasons with the same normalized ratio $\hat{\alpha}$, i.e., $\alpha_1 = \hat{\alpha} \cdot \alpha_{0, high}$ and $\alpha_0 = \hat{\alpha} \cdot \alpha_{0, low}$. As is shown in this figure, with fixed peak-to-base price ratio $\alpha$, the proposed system starts to become profitable at $u^B = 0.0081/(\text{kWh})$. With fixed base-hour price, the proposed system starts to become profitable at 0.7 of the original price difference, which means $u^PK = 0.2119/(\text{kWh})$ in high season and $u^PK = 0.0769/(\text{kWh})$ in low season. The annual profits grow almost linearly with regard to both base-hour price and peak-to-base ratio.
2) Impact of Capital Cost of Electrical Energy Storage Elements: The capital cost variations of batteries affect only the APM problem solution since the DCR problem does not take battery costs as inputs. Lead-acid batteries are among the oldest and most developed rechargeable battery technologies: The unit price of lead-acid batteries has been stable due to its being a mature battery technology for a few decades [8], [36]. Unlike lead-acid batteries, Li-ion battery technology is still developing despite its wide application in EVs and portable devices [40]. Fig. 10 presents how the capital costs of the two battery prices affect the annual profits. We find that the Li-ion battery unit price $p_{Li}$ has more influence than the lead-acid battery unit price $p_{Pb}$. Given fixed $p_{Li}$, the HEES system has averagely 37.96% higher annual profit if $p_{Pb}$ is reduced by half from $100/(kWh)$ to $50/(kWh)$ for ($1000, 50$ L), and 8.76% higher for ($3000, 50$ L). However, if $p_{Pb}$ is fixed and the Li-ion battery capital cost is reduced by half, e.g., comparing the HEES system with $200/(kWh)$ $p_{Li}$ and with $400/(kWh)$ $p_{Li}$, the former achieves averagely 66.12% higher annual profits for ($1000, 50$ L), and 130.19% higher for ($3000, 50$ L).

In order to show how the unit price ratio of Li-ion batteries and lead-acid batteries affects the HEES system design results, we adopt the degree of hybridization (DOH) as the capacity ratio of the main energy storage bank to the energy buffer, i.e., $\rho = (Q_{Li}/Q_{Pb})$. We assume fixed lead-acid battery unit price $[880/(kWh)]$, and take the ratio of Li-ion battery unit price and lead-acid battery unit price as a variable, i.e., $\beta = (p_{Li}/p_{Pb})$. Currently, the $\beta$ value is around 5, and is predicted to be around 2 to 3 in 2020 [40]. Fig. 11 shows the annual profit and the reciprocal\(^3\) of DOH ($1/\rho = (Q_{Pb}/Q_{Li})$) as functions of $\beta$. An intuitive conjecture is that lower $\beta$ value (Li-ion batteries getting cheaper) leads to optimization results of higher $\rho$, i.e., a larger energy buffer bank, which is proved in Fig. 11. It is also shown that if $\beta$ value is below a certain threshold [equal or less than 2 in Fig. 11(a) and 3 in Fig. 11(b)], the optimal solution is a Li-ion battery-only EES system.

3) Impact of Discretization in Bank Sizing: In practice, sizing battery banks is a discrete problem rather than a continuous problem. The proposed design approach might end up in a result that needs to be rounded up to the nearest integer number of energy storage elements. We show that this discretization step leads to negligible error in cost estimation in the following way: we scan $\pm 5\%$ range of the optimal Li-ion battery capacity $Q_{Li}$. For each $Q_{Li}$ value, there is a maximum $Q_{Pb}$ value determined by the budget limit. Since 100% to 105% $Q_{Pb}$ range exceeds the budget, we explore the annual saving profits from 95% to 100% of the maximum $Q_{Pb}$.

Fig. 12 shows the annual profits gained with battery specifications in Table V and $3000$ budget and $50$ L system volume, as a function of battery bank capacities. Unlike the Li-ion battery capacity $Q_{Li}$, values of $Q_{Pb}$ are shown as percentage since they vary for different $Q_{Li}$ values. The optimal Li-ion battery and lead-acid battery capacities are 430.2 and 199.7 Ah, respectively. The figure indicates that with given budget and volume constraint, the annual profit is unimodal with regard to battery capacities. A 5% difference in both bank’s capacities results in less than 5% degradation in the amortized annual profit. Therefore, the discretization in sizing battery banks does not impose additional difficulties to the proposed design approach.

4) Impact of Battery Model Accuracy: We use the simulation platform SIMES to conduct the evaluation of DCR to improve the accuracy of the results as mentioned in Section V-D. Since there are a large number of parameters in the detailed battery models adopted by SIMES, we choose the most important ones and test the changes in the final annual profit result, namely the battery internal resistance, the open circuit voltage (OCV) of batteries, and Peukert’s constants. Table VII lists the results for the HEES system with 18650 under constraints of $3000$ budget and $50$ L system volume and the Con Edison pricing policy. The parameters are simultaneously changed for both Li-ion batteries and lead-acid batteries. We can see that the proposed scheme is able to alleviate the 5% performance degradation in parameters (i.e., increase on internal resistance, decrease in OCVs, and increase

\(^3\)We use reciprocal of DOH because $Q_{Pb}$ might be zero.

![Fig. 10. Sensitivity analysis of battery capital costs. (a) $B = 1000$ and $V = 50$. (b) $B = 3000$ and $V = 50$.](image1)

![Fig. 11. Annual profit and DOH as a function of Li-ion to lead-acid battery unit price ratio. (a) $B = 1000$ and $V = 50$. (b) $B = 3000$ and $V = 50$.](image2)

![Fig. 12. Annual profit as a function of battery capacities.](image3)
TABLE VII
INFLUENCES ON ANNUAL PROFITS BY THE PARAMETERS IN BOTH LI-ION BATTERIES AND LEAD-ACID BATTERIES

<table>
<thead>
<tr>
<th>Battery Parameters</th>
<th>Change</th>
<th>Annual Profit $A$</th>
<th>A Decrease</th>
</tr>
</thead>
<tbody>
<tr>
<td>Internal Resistance</td>
<td>+5%</td>
<td>$114.7$</td>
<td>-2.1%</td>
</tr>
<tr>
<td>OCVs</td>
<td>-5%</td>
<td>$122.5$</td>
<td>-3.9%</td>
</tr>
<tr>
<td>$\eta^{Li}_{o} \eta^{P}$</td>
<td>+5%</td>
<td>$114.5$</td>
<td>-2.2%</td>
</tr>
<tr>
<td>(All the above)</td>
<td></td>
<td>$107.7$</td>
<td>-8.03%</td>
</tr>
</tbody>
</table>

on Peukert’s constants) for both battery banks. For example, the 5% OCV decrease in both battery banks directly leads to a 5% decrease of battery output power, which means a straight 5% reduction in energy savings without optimization. The proposed scheme alleviates the performance degradation to 3.9% by optimizing control and design decisions. If the three parameters concurrently get worsened, the proposed scheme yields an 8% decrease in the final annual profits, compared to $117.1 shown in Table VI. This indicates that the proposed HEES design method is able to mitigate the changes in battery models to prevent a small change in the battery parameters from invalidating the whole design and profit estimation.

VIII. CONCLUSION
One promising application of HEES systems is to deploy grid-connected HEES for residential users to save their electricity bills. Their economic viability in such applications depends on various input variables such as electricity pricing, energy storage element cost, etc. This paper presents an optimal grid-connected dual-bank HEES system design and control mechanism to maximize the profits for residential users, and provides a sensitivity analysis on all the factors that have major impacts on the potential profits. We propose a systematic two-step design framework: 1) the first step derives the energy buffering strategy that manages the charge and discharge currents of the banks to maximize daily energy savings and 2) the second step is to determine the system specifications (the energy storage element capacities, etc.) to maximize the amortized annual profits. Simulation results demonstrate that the proposed HEES system achieves an annual profit $118 based on a typical time-of-day electricity pricing and current storage element unit prices for a budget of $3000 and system volume of 50 L. The sensitivity analysis is conducted on four aspects, namely different electricity pricing policies, changing capital costs of energy storage elements, discretization in battery bank sizing, and battery model accuracy. We find that the actual profit of HEES systems is most sensitive to the electricity rates and the capital cost of the energy storage elements in the energy buffer bank. With rising electricity rates and decreasing energy storage element cost, the proposed grid-connected residential HEES system can achieve even higher profits.

REFERENCES


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